Rekurencja

Uwzględniamy:

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\begin{aligned} T_{n+1}(x) &= \cos[(n+1)\theta] = \cos(n\theta + \theta) \\ &= \cos(n\theta)\cos(\theta) - \sin(n\theta)\sin(\theta) \\ T_{n-1}(x) &= \cos[(n-1)\theta] = \cos(n\theta - \theta) \\ &= \cos(n\theta)\cos(\theta) + \sin(n\theta)\sin(\theta) \end{aligned}$$

Dodajemy $T_{n+1}(x)$ i $T_{n-1}(x)$

$$T_{n+1}(x) + T_{n-1} = 2\cos(n\theta)\cos(\theta) = 2x\,T_n(x)$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \qquad n \geq 1$$

NP.

$$T_2(x) = 2xT_1 - T_0 = 2x \cdot x - 1 = 2x^2 - 1$$

$$T_3(x) = 2xT_2 - T_1 = 2x(2x^2-1) - x = 4x^3 - 3x$$

$$T_4(x) = 2xT_3 - T_2 = 2x(4x^3-3x) - 2x^2 + 1$$

$$= 8x^4 - 6x^2 - 2x^2 + 1 = 8x^4 - 8x^2 + 1$$